NONCOMMUTATIVE METHODS IN TOPOLOGY AND GEOMETRY - ABSTRACTS

JUNE 1-5, 2015

COURSES

Ulrich Bunke. A secondary invariant

In this minicourse I will introduce an invariant of a pair of a framed foliation and a bundle with a flat partial connection in the direction of the foliation. After choosing additional geometric structures this invariant will be defined as a combination of spectral invariants of twisted Dirac operators and characteristic forms and their transgressions. I will recall some background one these objects and verify that the invariant is well-defined independently of the additional choices.
I will explain how this invariant generalizes and combines the ρ-invariant of a flat bundle, Adams e-invariant of stable homotopy classes, and secondary characteristic classes considered in foliation theory. I then move on and give an interpretation of the invariant in terms of differential K-theory. Finally, I will discuss some conjectures which interpret this invariant through constructions with algebraic K-theory and cyclic cohomology of the algebra of smooth functions.

Paolo Piazza. Geometric surgery sequences, Dirac operators and K-Theory

The goal of these lectures is to give a rapid introduction to coarse index theory and to its use in the problem of mapping geometric surgery sequences to K-Theory and, more precisely, to the Higson-Roe surgery sequence. This will involve primary (i.e. index theoretic) and secondary invariants of Dirac-type operators; the secondary invariants extend to a K-theoretic context the classic rho-invariants of Atiyah-Patodi-Singer and Cheeger-Gromov.
The main focus of the lectures will be on the definition and the interplay of these invariants and on the way they enter in the problem of making the transition from geometry to K-theory. This is joint work with Thomas Schick.

Georges Skandalis. Groupoids and pseudodifferential calculus

TALKS

Sara Azzali. Bivariant K-theory with $\mathbb{R}/\mathbb{Z}$-coefficients and rho classes of unitary representations, part 1

We review Atiyah-Patodi-Singer's class associated with a unitary flat vector bundle in the K-theory with $\mathbb{R}/\mathbb{Z}$-coefficients of a closed manifold. We show how Atiyah's theorem for covering spaces can be used as an ingredient of the construction of this class. We introduce the bivariant K-theory with real and $\mathbb{R}/\mathbb{Z}$-coefficients. (Joint work with P. Antonini and G. Skandalis)

Paolo Antonini. Bivariant K-theory with $\mathbb{R}/\mathbb{Z}$-coefficients and rho classes of unitary representations, part 2

Atiyah's L^2 index theorem for covering spaces expresses the triviality of the group trace on the index class in $K_0(C^*\Gamma)$. For a C^*-algebra endowed with a group action, we examine general cases for which the same triviality of the trace occurs. We call this a K-theoretically free and proper Γ-algebra (KFP). The definition uses the relation between traces on C^*-algebras and real KK-theory explained in part 1. We exhibit natural classes of algebras satisfying property (KFP).
Given a KFP algebra and unitary representation α of the group Γ, we construct a canonical rho class in KK-theory with $\mathbb{R}/\mathbb{Z}$-coefficients. This construction indeed generalizes the Atiyah-Patodi-Singer class of α. We

show that free and proper Γ-algebras (in the sense of Kasparov) have the (KFP) property. (Joint work with S. Azzali and G. Skandalis)

Teodor Banica. Liberation questions in noncommutative geometry

Given a real or complex algebraic manifold, the liberation problem is that of relaxing the commutation relations $ab = ba$ between its standard coordinates. This is reasonably understood in the compact Lie group case, where, under suitable assumptions, the relations $ab = ba$ can be replaced by the anticommutation relations $ab = -ba$ ("twisting"), or $abc = cba$ ("half-liberation"), or no relations at all ("full liberation"). I will discuss how these quantum group ideas apply to the general case, with a few basic examples.

Tathagata Banerjee. Coarse geometry for noncommutative spaces via compactifications

In this talk we shall discuss topological coarse structures coming from compactifications of a locally compact Hausdorff space. With an extended notion of coarse maps, we shall discuss the possibility of looking at coarse structures for a noncommutative space in terms of their unitalizations. Finally we discuss Rieffel deformation of unital C^*-algebras as specific examples for our theory. In particular this would give us a "coarse-equivalence" between the Moyal plane and the classical phase-space as is expected in Physics.

Karsten Bohlen. Boutet de Monvel's calculus via groupoid actions

We present a Boutet de Monvel calculus for general pseudodifferential boundary value problems defined abstractly via a groupoid correspondence. Then we discuss how such a calculus can be applied to the study of index theory on a broad class of singular and non-compact manifolds, so-called Lie manifolds with boundary.

Robin Deeley. A geometric model for Higson-Roe's analytic surgery group, part 1

In their papers "Mapping surgery to analysis I, II, and III", Higson and Roe construct an analytic counterpart to the classical surgery exact sequence of Browder, Novikov, Sullivan and Wall. In joint work with Magnus Goffeng, we use the geometric model of K-homology due to Baum and Douglas to construct a "geometric" version of Higson and Roe's exact sequence. The main goal of this talk is the construction of the geometric exact sequence; it is obtained by applying a relative construction in geometric K-homology to the Baum-Connes assembly map. (No knowledge of classical surgery or Higson and Roe's analytic surgery exact sequence are required for the talk.)

Magnus Goffeng. A geometric model for Higson-Roe's analytic surgery group, part 2

This talk will be an independent continuation of the talk by Robin Deeley. We will discuss the isomorphism between the geometric model for the analytic surgery group with Higson-Roe's original model as well as other higher invariants on the geometric model, such as higher delocalized eta invariants. The geometric cycles for surgery allow for a delocalized Chern character taking values in a delocalized version of de Rham homology giving back the delocalized eta invariant in examples. Based on joint work with Robin Deeley.

Piotr Hajac. Non contractibility of compact quantum groups and index pairing for their non-reduced suspensions

Using the concept of an equivariant join $G \star G$ of a compact quantum group G with itself, we define the contractibility of G as the existence of a global section of the compact quantum principal bundle $G \star G$ over the non-reduced suspension SG. We unravel the pullback structure of finitely generated projective modules associated to $G \star G$, and make it fit the Milnor connecting homomorphism formula in K-theory of unital C^*-algebras. Then, taking advantage of the compatibility of the index pairing with the connecting homomorphisms of the Mayer-Vietoris six-term exact sequences for K-theory and K-homology (which is a manifestation of the associativity of the Kasparov product), we prove that $SU_q(2)$ is not contractible, i.e. that Pflaum's quantum instanton bundle $SU_q(2) \star SU_q(2)$ is not trivializable. Finally, we conjecture the non-contractibility of all non-trivial compact quantum groups, and explain how it fits the bigger picture of noncommutative Borsuk-Ulam-type conjectures. (Based on joint work with P. F. Baum, L. Dabrowski, T. Hadfield, R. Matthes and E. Wagner.)

James Heitsch. Chern-Connes character of transverse Dirac operators

Seunghun Hong. Duflo Isomorphism and Chern-Weil Theory

We explain how the distributional index of an operator on a principal G-manifold P that is obtained by lifting a Dirac operator on P/G can serve as a link between the Duflo isomorphism and Chern-Weil forms.

Tomasz Maszczyk. Cyclic homology and quantum orbits (joint with Serkan Sutlu)

A natural isomorphism between the cyclic object computing the relative cyclic homology of a homogeneous quotient-coalgebra-Galois extension, and the cyclic object computing the cyclic homology of a Galois coalgebra with SAYD coefficients is presented. The isomorphism can be viewed as the cyclic-homological counterpart of the Takeuchi-Galois correspondence between the left coideal subalgebras and the quotient right module coalgebras of a Hopf algebra. A spectral sequence generalizing the classical computation of Hochschild homology of a Hopf algebra to the case of arbitrary homogeneous quotient-coalgebra-Galois extensions is constructed. A Pontryagin type self-duality of the Takeuchi-Galois correspondence is combined with the cyclic duality of Connes in order to obtain dual results on the invariant cyclic homology, with SAYD coefficients, of algebras of invariants in homogeneous quotient-coalgebra-Galois extensions. The relation of this dual result with the Chern character, Frobenius reciprocity, and inertia phenomena in the local Langlands program, the Chen-Ruan-Brylinski-Nistor orbifold cohomology and the Clifford theory will be discussed.

Ralf Meyer. Actions of inverse semigroups on groupoids.

We define how an inverse semigroup actions on a topological groupoid by partial equivalences. From this, we construct an action of the inverse semigroup on a C^*-algebra by Hilbert bimodules (partial Morita equivalences). This is the same as a Fell bundle over the inverse semigroup. The motivating example is the action of a non-Hausdorff etale groupoid G on its own "space" of arrows. Here the non-Hausdorff space of arrows is modeled by an etale groupoid H using a covering of the arrow space by Hausdorff open subsets. The left or right multiplication action of the groupoid on its arrows is described through an action of the inverse semigroup $\mathrm{Bis}(G)$ of bisections of G on H. This induces an action of $\mathrm{Bis}(G)$ on $C^*(H)$ by Hilbert bimodules. These actions cannot be strictified to (untwisted) actions by automorphisms. They are prototypical examples of free and proper actions of G.

Victor Nistor. Analysis and index theory on noncompact spaces: operator algebras, groupoids, and applications

The main goal of my talk is to present an application of index theory to boundary value problems. More specifically, usingsome rather elementary index theory calculations in the spirit of the APS theory, I will obtain a well-posedness result for the Neumann problem on polygons. We check this result using computer tests. I will then use this result to motivate an approach to analysis on singular and non-compact spaces using groupoids and Lie manifolds.

Raphael Ponge. Noncommutative geometry, equivariant cohomology, and conformal invariants.

In this talk I shall explain how to apply tools of noncommutative geometry to study conformal geometry. We will present two main results. The first result is a reformulation of the local index formula in conformal-diffeomorphism invariant geometry. This builds on earlier work of Connes-Moscovici. The second main result is the construction of a whole new family of conformal invariants and their explicit computation in terms of equivariant characteristic classes. This involves the computation by means of an explicit quasi-isomorphism of the cyclic homology of the crossed-product of the algebra of smooth functions on a closed Riemannian manifold with an arbitrary discrete group of isometries.

Hessel Posthuma. Index theory for Lie algebroids and integration

I will introduce two index theorems for Lie algebroids. The first one uses an integration of the Lie algebroid to a Lie groupoid and generalizes the Atiyah-Singer index theorem in various directions. The second index theorem depends only on the germ of such an integration around the units. The point is that the first type of integration to Lie groupoids is subject to certain obstructions discovered by Crainic-Fernandes, whereas the second type of "formal integration" is unobstructed. The second index theorem should therefore be viewed as an extension of the first to (possibly) nonintegrable Lie algebroids.

Anton Savin. Uniformization problem for elliptic G-operators and KK-theory (joint work with Boris Sternin)

Elmar Schrohe. Fourier integral operators on manifolds with boundary and the Atiyah-Weinstein index

Jianchao Wu. The amenability dimension and company

We showcase a number of recently emerged concepts of dimensions defined for topological dynamical systems, such as the amenability dimension. These dimensions turn out to be of great use to the study of the Baum-Connes conjecture and the Farrell-Jones conjecture, as well as the nuclear dimensions of crossed product C^*-algebras, which is an important regularity property in the classification program of simple separable amenable C^*-algebras. They have close relations with the Rokhlin dimension defined for C^*-dynamics, and often take finite values under reasonable assumptions. This talk includes work in collaboration with Hirshberg, Szabo, Winter and Zacharias as well as further recent developments.

Rudolf Zeidler. Positive scalar curvature and product formulas for secondary index invariants

We will discuss secondary invariants associated to metrics of uniformly positive scalar curvature (PSC) on complete spin manifolds. The main application of such invariants is to distinguish PSC metrics (e.g. up to bordism or concordance). We exhibit product formulas for the higher rho-invariant of a PSC metric as well as for the higher relative index of two PSC metrics. We adopt the approach of Xie-Yu and use the K-theory of Yu's localization algebras as receptacles for secondary index invariants. The main technical novelty is that we use the localization algebras together with a certain description of K-theory for graded C^*-algebras due to Trout.

This formalism allows direct definitions of all the invariants we consider in terms of the functional calculus of the Dirac operator and enables us to give concise proofs of the product formulas. If time permits, we apply the product formulas to give a proof of a secondary partitioned manifold index theorem due to Piazza-Schick and show how it can be used to distinguish PSC metrics via certain submanifolds of codimension one and two.

Vito Zenobi. Lie Groupoids and Secondary Invariants

A pair of homotopy equivalent manifolds or a metric with positive scalare curvature on a Riemannian spin manifold are examples in which the analytic index vanishes (for the signature operator and the Dirac operator respectively). These situations give secondary invariants as lifting of K-homology classes to a certain K-group. We define these invariants in more complicated geometrical situations (manifolds with corners, foliations, etc...) encoded by a Lie groupoid as elements in the K-theory of the C^*-algebra of the adiabatic groupoid. These invariants apply to the study of the Structure Set (in the Surgery Exact Sequence) and the space of positive scalar curvature metrics of a Riemannian manifold.